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Mixed convection in non-Newtonian fluids along a vertical plate in porous media with surface mass transfer

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Nomenclature

f	=	dimensionless stream function
g	=	acceleration due to gravity
h	=	heat transfer coefficient
k	=	thermal conductivity
K	=	permeability for the porous medium
L	=	plate length
n	=	viscosity index
Nu	=	Nusselt number
Pe	=	Peclet number
q_w	=	wall heat flux
Ra	=	Rayleigh number
T	=	temperature
u,v	=	velocity components in x and y directions
U_∞	=	free stream velocity
v_o	=	velocity in the case of surface mass transfer
x,y	=	axial and normal co-ordinates
α	=	effective thermal diffusivity of porous medium
β	=	volumetric coefficient of thermal expansion
η	=	similarity variable
Θ	=	dimensionless temperature
ν	=	kinematic viscosity
ξ	=	mass transfer parameter
ρ	=	density of fluid
μ	=	consistency index for viscosity
τ_w	=	wall shear stress
χ	=	mixed convection parameter
ψ	=	stream function

Subscripts

w	=	wall conditions
∞	=	free stream conditions

Note: The symbols defined above are subject to alteration on occasion

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Introduction

The study of convective heat transfer from surfaces embedded in porous media has received considerable attention recently because of their numerous thermal engineering applications such as geothermal systems, oil extraction, thermal insulation and ground water pollution. Cheng and Minkowycz[1] presented similarity solutions for free convective heat transfer from a vertical plate in a fluid-saturated porous medium. Gorla and co-workers[2-4] solved the non-similar problem of free convective heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrarily varying surface temperature or heat flux. The mixed convection from surfaces embedded in porous media was studied by Minkowycz *et al.*[5] and Ranganathan and Viskanta[6]. Hsieh *et al.*[7] presented non-similar solutions for mixed convection in porous media. All these studies were concerned with Newtonian fluid flows. A number of industrially important fluids, including fossil fuels which may saturate underground beds, display non-Newtonian behaviour. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate.

Chen and Chen[8] presented similarity solutions for free convection of non-Newtonian fluids over vertical surfaces in porous media. Nakayama and Koyama[9] studied the natural convection over a non-isothermal body of arbitrary shape embedded in a porous medium.

The present work has been undertaken in order to analyse the mixed convection from a vertical plate in non-Newtonian fluid saturated porous media. The effect of surface injection or suction is taken into account. The governing equations are first transformed into a dimensionless form and the resulting non-similar set of equations are solved by a finite difference method. Numerical results are presented for some representative values of the viscosity index.

Analysis

Let us consider mixed convection from a permeable vertical plate embedded in a non-Newtonian fluid-saturated porous medium, in the presence of surface injection or suction at a uniform velocity v_0. The co-ordinate system and flow model are shown in Figure 1. We consider the Darcy model assuming low velocity and porosity. Invoking the Boussinesq approximation, the governing boundary layer equations may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u^n = U_\infty^n + \frac{K}{\mu} [\rho g \beta (T - T_\infty)] \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

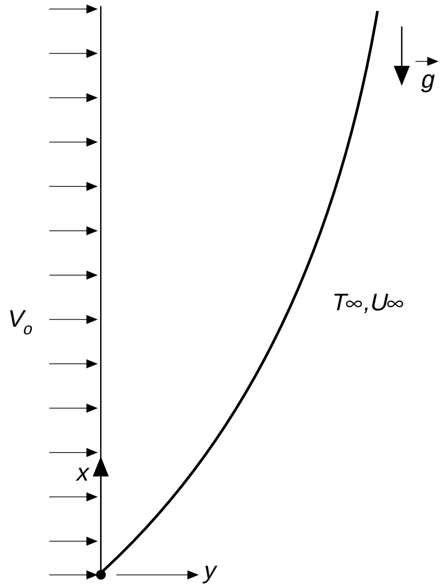

Figure 1.
Co-ordinate system and flow development

The appropriate boundary conditions are given by

$$
\begin{aligned}
y = 0: \quad & v = v_0, \quad T = T_w (constant) \\
y \to \infty: \quad & u = U_\infty, \quad T = T_\infty
\end{aligned}
\tag{4}
$$

In the above equations, $u(x,y)$ and $v(x,y)$ are the velocity components in x and y directions; $T(x,y)$ the temperature; ρ, μ and β the density, consistency index for viscosity and thermal expansion coefficient of the fluid; g the acceleration due to gravity; K and α the permeability and effective thermal diffusivity of the porous medium; and n the viscosity coefficient for the fluid. The analysis is performed for the buoyancy assisting flow condition. Therefore, for an upward forced flow, we have $T_w > T_\infty$ and for downward flow, $T_w < T_\infty$. We now define a stream function ψ such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. The continuity equation (1) is then automatically satisfied.

Proceeding with the analysis, we define:

$$\eta = \frac{y}{x}\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right]$$

$$\xi = \frac{v_o x}{\alpha}\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right]^{-1}$$

$$\psi = \alpha\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right] f(\xi, \eta)$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}$$

$$\chi = [1 + (Ra_x/Pe_x)^{\frac{1}{2}}]^{-1} \tag{5}$$

$$Pe_x = \frac{U_\infty x}{\alpha}$$

$$Ra_x = \frac{x}{\alpha} [\frac{\rho K g\beta \ (T_w-T_\infty)}{\mu}]^{\frac{1}{n}}$$

On substituting expressions (5) into equations (2) and (3) we have:

$$n(f')^{n-1} f'' = \chi^{2(n-1)} (1-\chi)^2 \theta' \tag{6}$$

$$\theta'' + \frac{f\theta'}{2} = \frac{\xi}{2} [f' \frac{\partial\theta}{\partial\xi} - \theta' \frac{\partial f}{\partial\xi}] \tag{7}$$

The transformed boundary conditions are given by

$$\begin{aligned} \eta = 0: \quad & f(\xi,0) + \xi \frac{\partial f}{\partial \xi}(\xi,0) = -2\xi, \quad \theta(\xi,0) = 1 \\ \eta \to \infty: \quad & f'(\xi,\infty) = \chi^2, \quad \theta(\xi,\infty) = 0 \end{aligned} \tag{8}$$

The primes in the previous equations denote partial differentiation with respect to η only. We note that $\chi = 0$ corresponds to pure natural convection whereas $\chi = 1$ corresponds to pure forced convection. ξ is positive for injection and negative for suction. In practical applications, it is usually the surface characteristics such as friction factor and Nusselt number that are of importance.

Defining the local Nusselt number $Nu_x = \frac{hx}{k_f}$ where $h = q_w/(T_w - T_\infty)$ we have

$$Nu_x = -(Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}) \ \theta'(\xi,0) \tag{9}$$

Numerical scheme

The numerical scheme to solve equations (6) and (7) adopted here is based on a combination of the following concepts:

(1) The boundary conditions for $\eta = \infty$ are replaced by

$$f'(\xi,\eta_{max}) = \chi^2, \quad \theta(\xi,\eta_{max}) = 0 \tag{10}$$

where η_{max} is a sufficiently large value of η at which the boundary conditions (8) are satisfied. η_{max} varies with the value of η. In the present work, we set $\eta_{max} = 25$.

(2) The two-dimensional domain of interest (ξ, η) is discretized with an equispaced mesh in the ξ-direction and another equispaced mesh in the η-direction.

(3) The partial derivatives with respect to η and ξ are evaluated by the central difference approximations.

(4) Two iteration loops based on the successive substitution are used because of the non-linearity of the equations.

(5) In each inner iteration loop, the value of ξ is fixed while each of the equations (6) and (7) is solved as a linear second order boundary value problem of ODE on the η-domain. The inner iteration is continued until the non-linear solution converges for the fixed value of ξ.

(6) In the outer iteration loop, the value of ξ is advanced from –2 to 2. The derivatives with respect to ξ are updated after every outer iteration step.

In the inner iteration step, the finite difference approximation for equations (6) and (7) is solved as a boundary value problem. We consider equation (6) first. By defining $U = \phi$, equation (6) may be written in the form

$$a_1\phi'' + b_1\phi' + c_1\phi = S_1 \tag{11}$$

where

$$\begin{aligned} a_1 &= n\,|\phi'|^{n-1} \\ b_1 &= c_1 = 0 \\ s_1 &= \chi^{2(n-1)}\,(1-\chi)^2\,\theta' \end{aligned} \tag{12}$$

The coefficients a_1, b_1 c_1 and the source term in equation (11) in the inner iteration step are evaluated by using the solution from the previous iteration step. Equation (11) is then transformed to a finite difference equation by applying the central difference approximations to the first and second derivatives. The finite difference equations form a tridiagonal system and can be solved by the tridiagonal solution scheme.

Equation (7) is also written as a second-order boundary value problem similar to equation (12), namely:

$$a_2\theta'' + b_2\theta' + c_2\theta = s_2 \tag{13}$$

where

$$a_2 = 1$$
$$b_2 = \frac{U}{2}$$
$$c_2 = 0$$
$$s_2 = \frac{\xi}{2}\left[\phi' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial \Phi}{\partial \xi}\right] \qquad (14)$$

The numerical results are affected by the number of mesh points in both directions. To obtain accurate results, a mesh sensitivity study was performed. After some trials, in the η-direction 190 mesh points were chosen whereas in the ξ-direction, 41 mesh points were used.

Results and discussion

Numerical results for $\Theta'(\xi, 0)$ are tabulated in Table I to include $-2 < \xi < 2$. The velocity and temperature profiles are displayed in Figures 2-7 for various values of the mass transfer parameter, ξ. The momentum and thermal boundary layer thicknesses increase as ξ increases in the case of injection (ξ, 0) and decrease with increasing suction (ξ, 0). The velocity and temperature profiles tend to become box type as n and ξ approach a value of 2.

Figures 8 and 9 display the variation of Nusselt number with χ for the cases of suction and injection. Figure 8 shows that increasing values of suction parameter ξ results in augmented surface heat transfer rate. As the viscosity index n increases, we notice that the Nusselt number decreases. Figure 9 shows that as the values of the injection parameter ξ increase, the Nusselt number decreases. Increasing values of the viscosity index also decrease the Nusselt number in the case of injection.

	ξ, = 2.0	–1.5	–1.0	–0.5	0.0	0.5	1.0	1.5	2.0
n = 0.5									
χ = 0	2.00386	1.50377	1.00154	0.49600	0.50935	0.18076	0.09878	0.05012	0.03121
χ = 0.5	2.00964	1.53385	1.10192	0.74085	0.5348	0.27401	0.149699	0.07195	0.03383
χ = 1.0	2.05106	1.60800	1.20332	0.85152	0.5642	0.34945	0.19894	0.10278	0.04753
n = 1.0									
χ = 0.5	2.0016	1.5106	1.0491	0.6543	0.3603	0.1719	0.0704	0.0242	0.0069
n = 2.0									
χ = 0.5	2.00056	1.50550	1.03325	0.62278	0.3184	0.13244	0.04419	0.01156	0.0023

Table I.
Values of $-\Theta'(\xi, 0)$

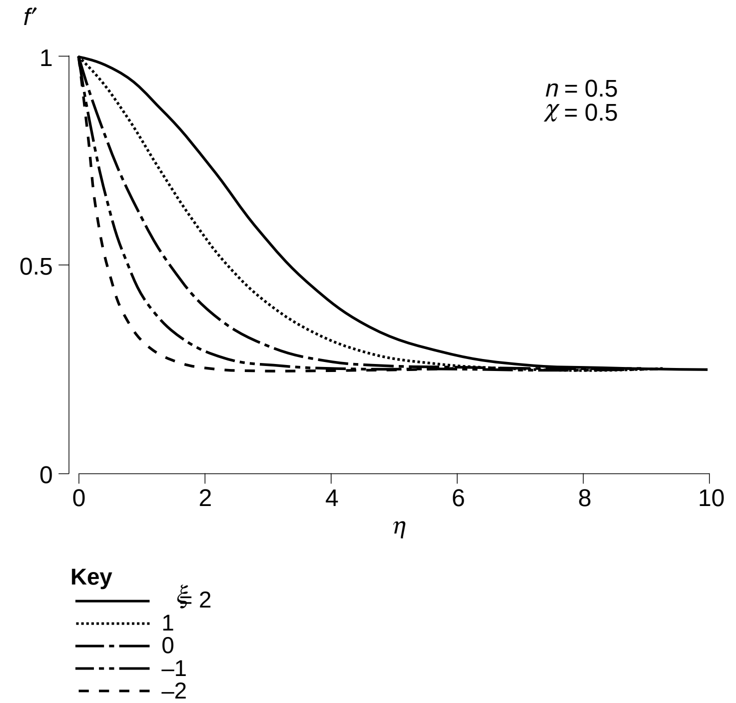

Figure 2.
Velocity-distribution
(n = 0.5)

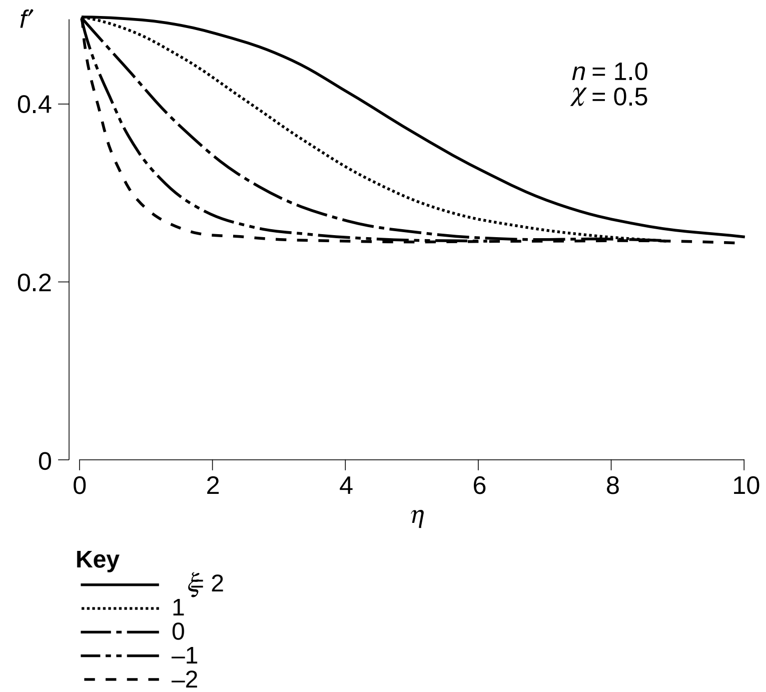

Figure 3.
Velocity-distribution
(n = 1)

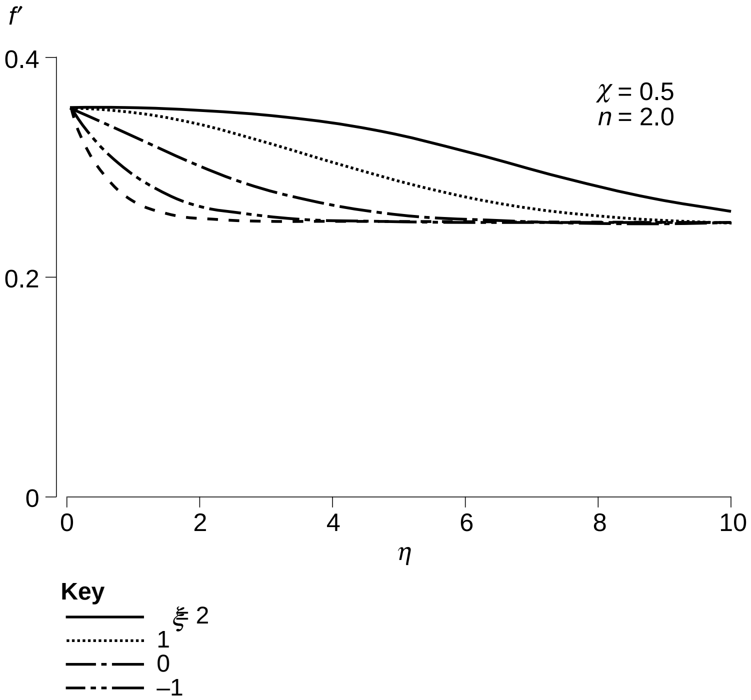

Figure 4.
Velocity-distribution
(n = 2)

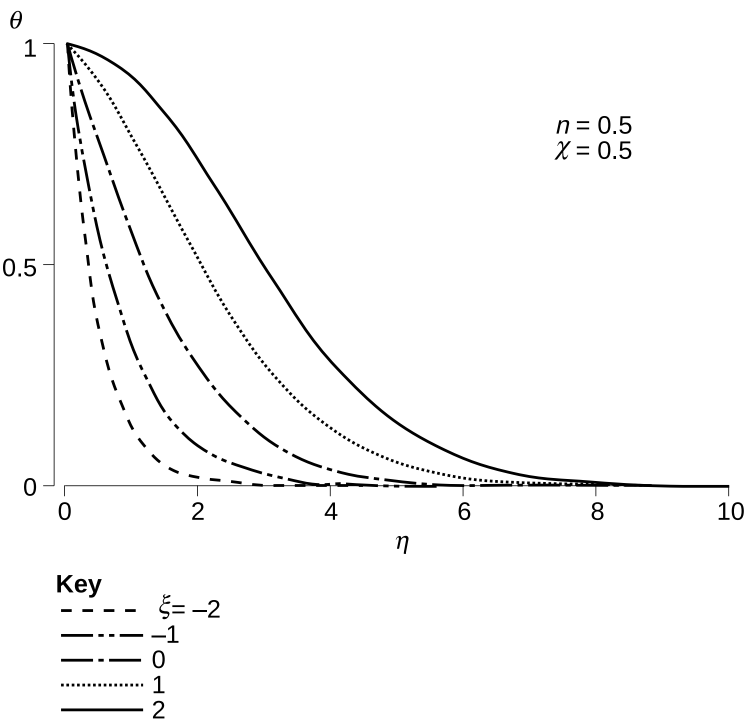

Figure 5.
Temperature-
distribution (n = 0.5)

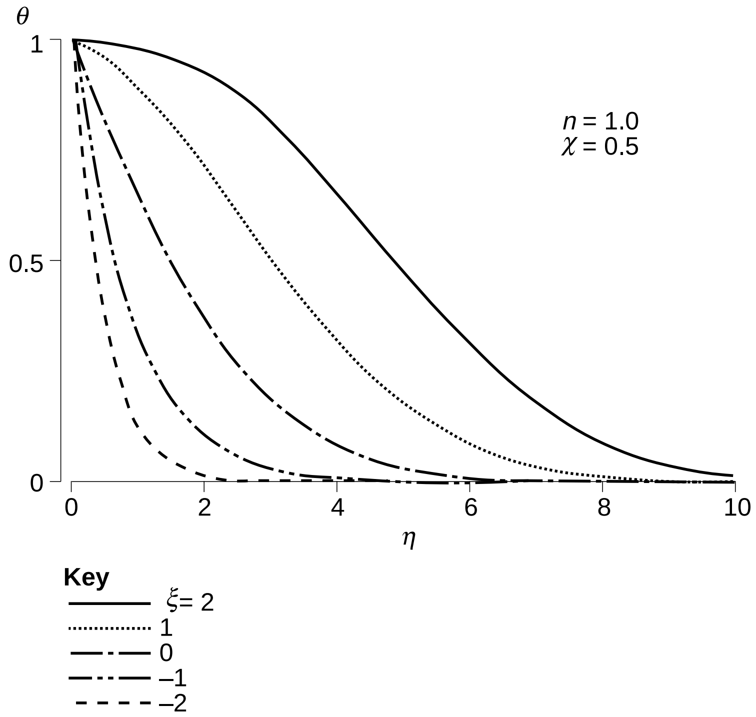

Figure 6.
Temperature-distribution (n = 1)

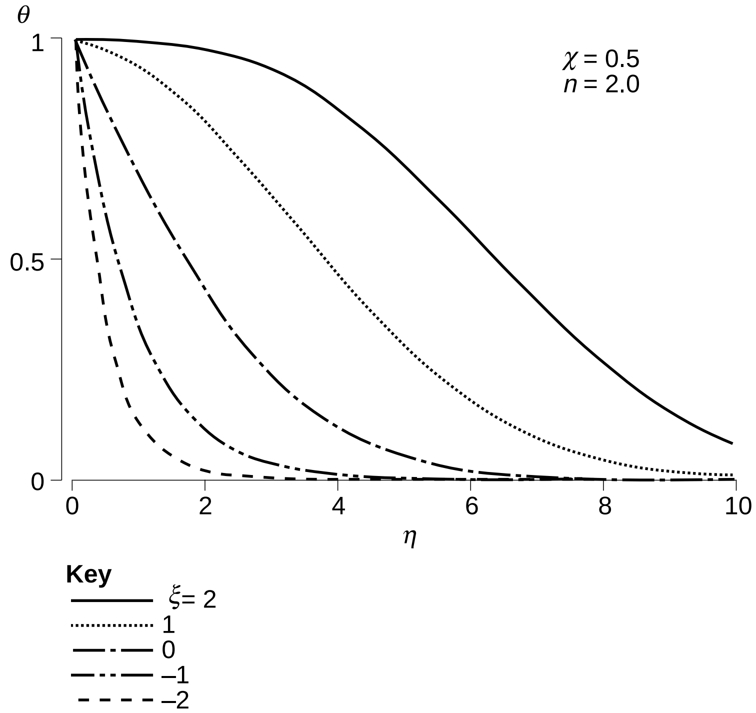

Figure 7.
Temperature-distribution (n = 2)

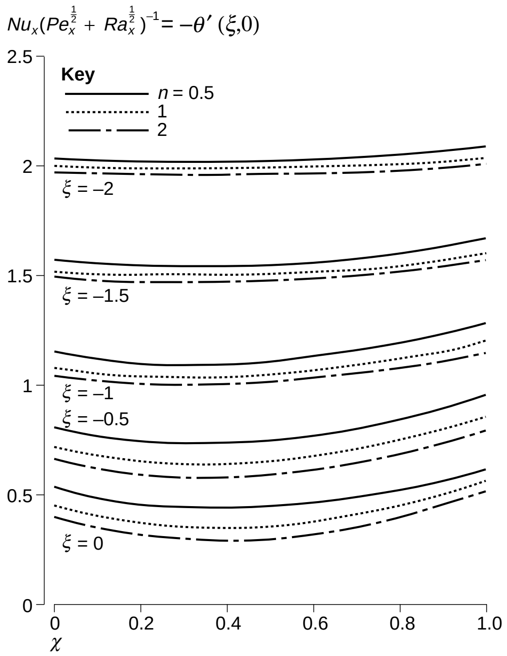

Figure 8.
Nusselt number versus χ (suction)

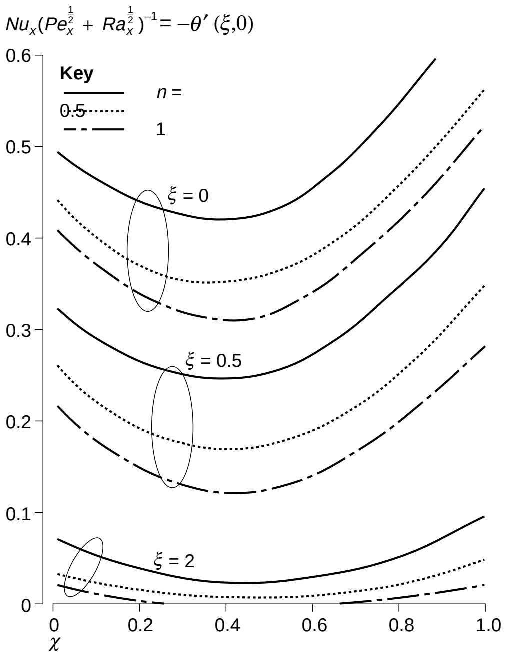

Figure 9.
Nusselt number versus χ (injection)

Concluding remarks

In this paper, we have presented a boundary layer analysis for the problem of mixed convection in non-Newtonian fluids along an isothermal vertical plate in a porous medium with surface mass transfer. Numerical results are presented for the velocity and temperature profiles as well as Nusselt number variation with the combined convection parameter χ. The cases of suction and injection have been examined as have the influence of the surface mass transfer and the viscosity index on the surface heat transfer rate.

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